Pearson Edexcel GCE

Thursday 20 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **6690/01**

Decision Mathematics D2

Advanced/Advanced Subsidiary

You must have:
D2 Answer Book

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** on the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the D2 answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.
- Do not return the question paper with the answer book.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Write your answers in the D2 answer book for this paper.

1.

	A	B	C	D	E	F
A	–	53	47	39	35	40
B	53	–	32	46	41	43
C	47	32	–	51	47	37
D	39	46	51	–	36	49
E	35	41	47	36	–	42
F	40	43	37	49	42	–

The table above shows the least distances, in km, between six towns, A, B, C, D, E and F. Jas needs to visit each town, starting and finishing at D, and wishes to minimise the total distance she travels.

(a) Starting at D, use the nearest neighbour algorithm to obtain an upper bound for the length of the route. You must state your route and its length.

(2)

(b) Starting by deleting D, and all of its arcs, find a lower bound for the route length.

(3)

(Total 5 marks)

2. Table 1 shows the cost, in pounds, of transporting one unit of stock from each of three supply points, A, B and C, to each of four demand points, 1, 2, 3 and 4. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution is required.

	1	2	3	4	Supply
A	17	20	23	14	25
B	16	15	19	22	29
C	19	14	11	15	32
Demand	28	17	23	18	

Table 1

Table 2 shows an initial solution given by the north-west corner method.

	1	2	3	4
A	25			
B	3	17	9	
C			14	18

Table 2

(a) Taking A4 as the entering cell, use the stepping-stone method to find an improved solution. Make your route clear.

(2)

(b) Taking the most negative improvement index to indicate the entering cell, use the stepping-stone method once to obtain an improved solution. You must make your method clear by stating your shadow costs, improvement indices, route, entering cell and exiting cell.

(4)

(c) Determine whether your current solution is optimal, giving a reason for your answer.

(3)

(d) State the cost of your current solution.

(1)

(Total 10 marks)

3. Five friends have rented a house that has five bedrooms. They each require their own bedroom. The table below shows how each friend rated the five bedrooms, A, B, C, D and E, where 0 is low and 10 is high.

	A	B	C	D	E
Frank	5	0	7	3	4
Gill	5	3	8	10	1
Harry	4	3	7	9	0
Imogen	6	3	6	5	4
Jiao	0	2	7	3	2

Reducing rows first, use the Hungarian algorithm to obtain an allocation that maximises the total of all the ratings. You must make your method clear and show the table after each stage.

(Total 8 marks)

4. Eugene and Stephen play a zero-sum game. The pay-off matrix shows the number of points that Eugene scores for each combination of strategies.

	Stephen plays 1	Stephen plays 2	Stephen plays 3
Eugene plays 1	4	5	0
Eugene plays 2	–2	1	1
Eugene plays 3	–3	–4	3

(a) Find the play-safe strategies for each of Eugene and Stephen, and hence show that this zero-sum game does not have a stable solution.

(4)

(b) Suppose that Eugene knows that Stephen will use his play-safe strategy. Explain why Eugene should change from his play-safe strategy. You should state as part of your answer which strategy Eugene should now play.

(1)

(c) Formulate the game as a linear programming problem for Stephen. Define your variables clearly. Write the constraints as equations.

(7)

(Total 12 marks)

5. A linear programming problem in x, y and z is described as follows.

Maximise $P = 2x + 3y + z$

subject to
$$2y - 3z \leqslant 30$$
$$-3x + y + z \leqslant 60$$
$$x + 4y - z \leqslant 80$$

(a) Complete the initial tableau in the answer book for this linear programming problem.

(3)

(b) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the simplex algorithm to obtain a new tableau, T. Make your method clear by stating the row operations you use.

(5)

(c) Write down the profit equation given by T and state the values of the slack variables given by T.

(2)

The following tableau is obtained after further iterations.

Basic variable	x	y	z	r	s	t	Value
r	0	2	–3	1	0	0	30
s	0	13	–2	0	1	3	300
x	1	4	–1	0	0	1	80
P	0	5	–3	0	0	2	160

(d) Explain why no optimal solution can be found by applying the simplex algorithm to the above tableau.

(1)

(Total 11 marks)

6.

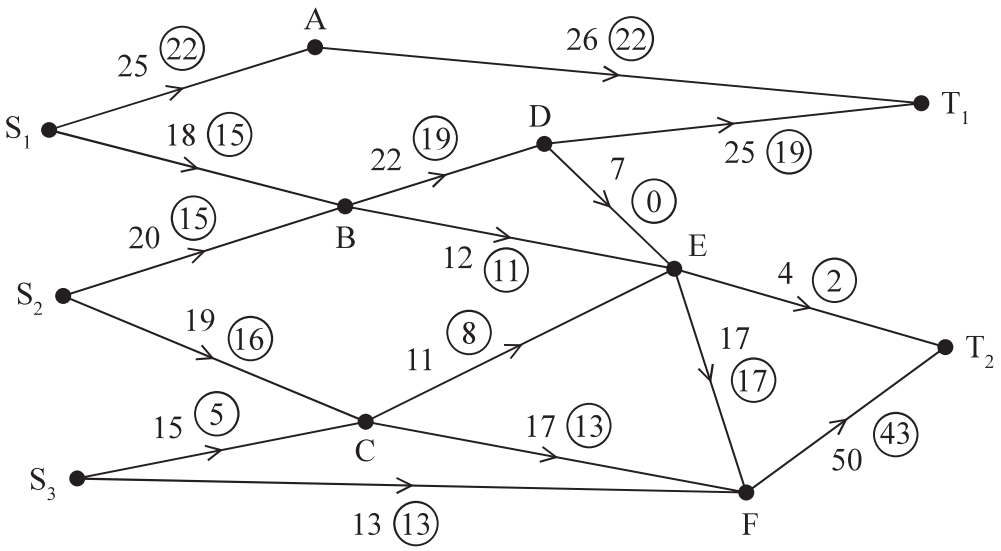

Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.

(1)

(b) (i) Add a supersource, S, and a supersink, T, and corresponding arcs to Diagrams 1 and 2 in the answer book.

(ii) Enter the flow value and appropriate capacity on each of the arcs you have added to Diagram 1.

(3)

(c) Complete the initialisation of the labelling procedure on Diagram 2 in the answer book by entering values along the new arcs from S to T, and along arcs S_1B and AT_1

(2)

(d) Hence use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

(4)

(e) Draw a maximal flow pattern on Diagram 3 in the answer book.

(2)

(f) Prove that your flow is maximal.

(2)

(Total 14 marks)

7. A company has purchased a plot of land and has decided to build four holiday homes, A, B, C and D, on the land at the rate of one home per year.

The company expects that the construction costs each year will vary, depending on which houses have already been constructed and which house is currently under construction. The expected construction costs, in thousands of pounds, are shown in the table below.

	Construction cost			
Already built	A	B	C	D
Nothing	25	35	35	30
A	–	45	50	55
B	50	–	45	55
C	60	45	–	50
D	65	55	60	–
A and B	–	–	40	35
A and C	–	55	–	30
A and D	–	55	60	–
B and C	45	–	–	50
B and D	65	–	35	–
C and D	55	40	–	–
A and B and C	–	–	–	65
A and B and D	–	–	55	–
A and C and D	–	45	–	–
B and C and D	55	–	–	–

In order to manage annual cash flow, the company needs to choose the order in which it builds the holiday homes so that the greatest expected annual cost of construction is as small as possible.

(a) Write down the type of dynamic programming problem that the company needs to solve.

(1)

(b) Use dynamic programming to complete the table in the answer book.

(11)

(c) Hence determine the order in which the four holiday homes should be built and the total expected cost of construction.

(3)

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel GCE

Centre Number | | | | | |

Candidate Number | | | | |

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Answer Book
Do not return the question paper with the answer book.

Total Marks

Turn over ▶

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P 5 5 8 6 2 A 0 1 1 6

Pearson

Leave blank

1.

	A	B	C	D	E	F
A	–	53	47	39	35	40
B	53	–	32	46	41	43
C	47	32	–	51	47	37
D	39	46	51	–	36	49
E	35	41	47	36	–	42
F	40	43	37	49	42	–

Leave blank

Question 1 continued

	A	B	C	D	E	F
A	–	53	47	39	35	40
B	53	–	32	46	41	43
C	47	32	–	51	47	37
D	39	46	51	–	36	49
E	35	41	47	36	–	42
F	40	43	37	49	42	–

Q1

(Total 5 marks)

Leave blank

2.

	1	2	3	4	Supply
A	17	20	23	14	25
B	16	15	19	22	29
C	19	14	11	15	32
Demand	28	17	23	18	

Table 1

You may not need to use all of these tables

	1	2	3	4	Supply
A					25
B					29
C					32
Demand	28	17	23	18	

	1	2	3	4	Supply
A					25
B					29
C					32
Demand	28	17	23	18	

	1	2	3	4	Supply
A					25
B					29
C					32
Demand	28	17	23	18	

	1	2	3	4	Supply
A					25
B					29
C					32
Demand	28	17	23	18	

Leave blank

Question 2 continued

	1	2	3	4	Supply
A					25
B					29
C					32
Demand	28	17	23	18	

	1	2	3	4	Supply
A					25
B					29
C					32
Demand	28	17	23	18	

	1	2	3	4	Supply
A					25
B					29
C					32
Demand	28	17	23	18	

	1	2	3	4	Supply
A					25
B					29
C					32
Demand	28	17	23	18	

	1	2	3	4	Supply
A					25
B					29
C					32
Demand	28	17	23	18	

(Total 10 marks)

Q2

Leave blank

3.

	A	B	C	D	E
Frank	5	0	7	3	4
Gill	5	3	8	10	1
Harry	4	3	7	9	0
Imogen	6	3	6	5	4
Jiao	0	2	7	3	2

You may not need to use all of these tables

	A	B	C	D	E
F					
G					
H					
I					
J					

	A	B	C	D	E
F					
G					
H					
I					
J					

	A	B	C	D	E
F					
G					
H					
I					
J					

Leave blank

Question 3 continued

	A	B	C	D	E
F					
G					
H					
I					
J					

	A	B	C	D	E
F					
G					
H					
I					
J					

	A	B	C	D	E
F					
G					
H					
I					
J					

	A	B	C	D	E
F					
G					
H					
I					
J					

Q3

(Total 8 marks)

Leave blank

4.

	Stephen plays 1	Stephen plays 2	Stephen plays 3
Eugene plays 1	4	5	0
Eugene plays 2	–2	1	1
Eugene plays 3	–3	–4	3

Leave blank

Question 4 continued

(Total 12 marks)

Q4

Leave blank

5. (a)

b.v.	x	y	z	r	s	t	Value
							30
							60
							80
							0

You may not need to use all of these tableaux

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

b.v.	x	y	z	r	s	t	Value	Row Ops
P								

Leave blank

Question 5 continued

(Total 11 marks)

Q5

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6. (a) Value of initial flow _____

(b) and (c)

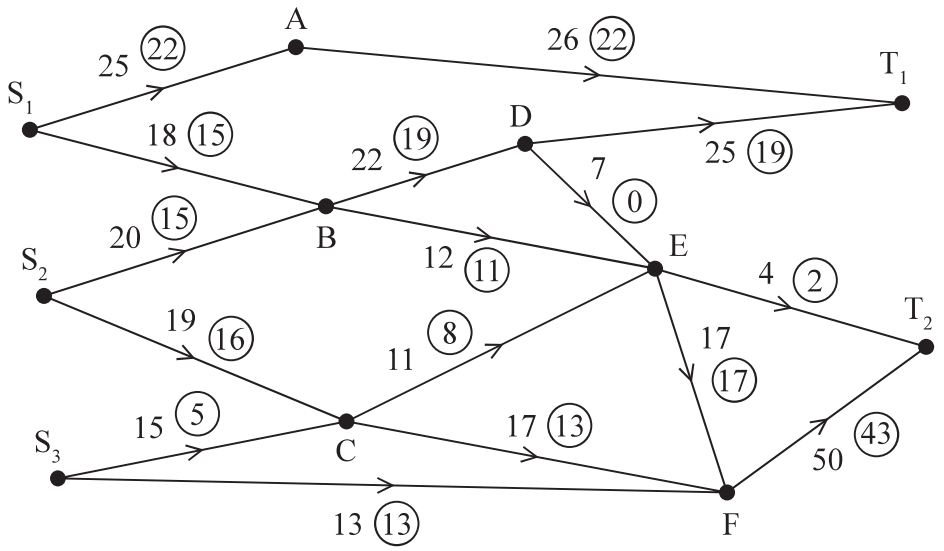

Diagram 1

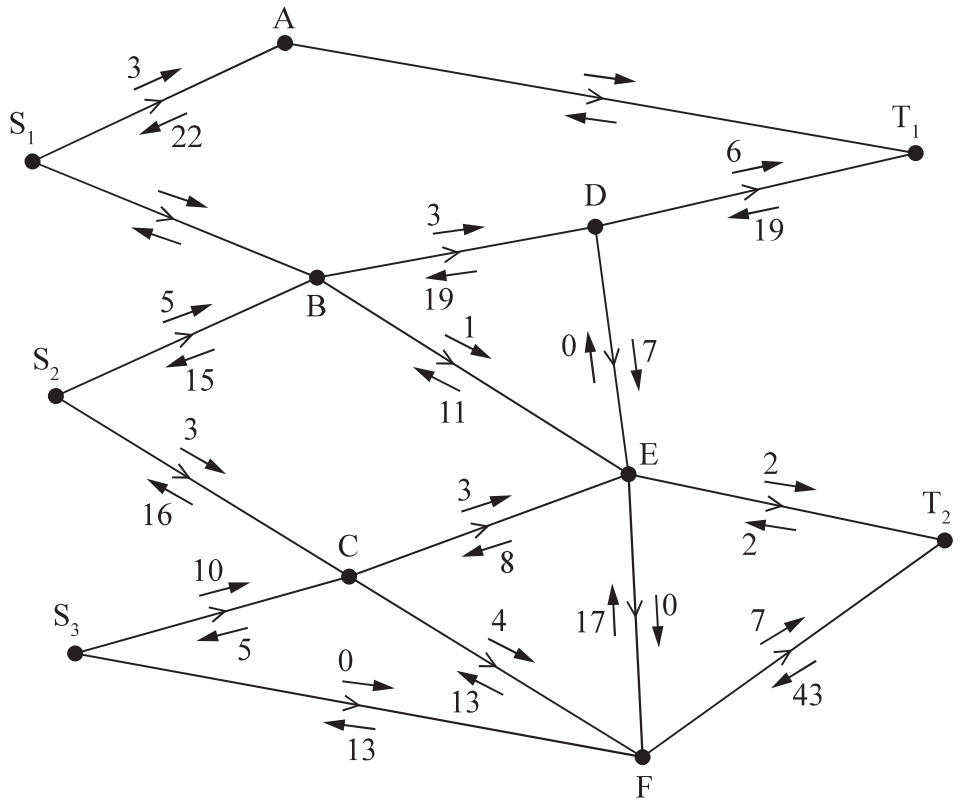

Diagram 2

Leave blank

Question 6 continued

(d)

(e)

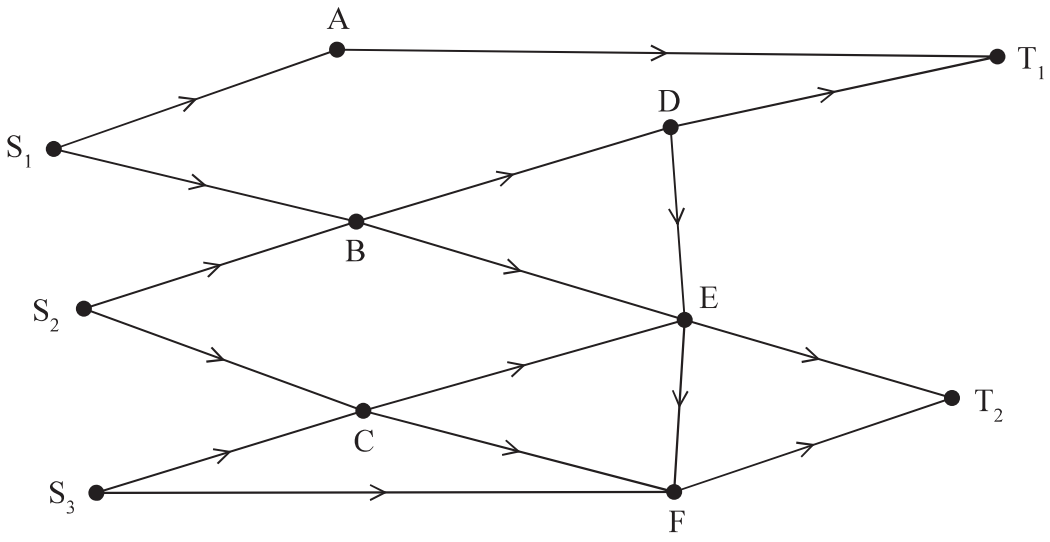

Diagram 3

(f)

Q6

(Total 14 marks)

Leave blank

7. (a)

Stage	State	Action	Dest.	Value
1	ABC	D	ABCD	65*

Leave blank

Question 7 continued

Stage	State	Action	Dest.	Value

Leave blank

Question 7 continued

Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END